Math 398 — Combinatorics — Winter, 2008

Professor: Dr S.P. Glasby

Course Information

Office:	Rm 119, Bouillon Hall.
Office Hrs:	At times outside office, and by appointment.
URL:	`http://www.cwu.edu/~glasbys/`
Lectures:	MTuThF 11:00 p.m., Lind 204.
Text:	Miklós Bóna, Introduction to Enumerative Combinatorics, McGraw Hill, 2007.
Assessment:	Test 1 (50%); Final Exam (50%).
Dates:	T1: Thu Feb 14; T2: E: Wed Mar 12, 8–10 a.m.?
Safari:	`http://portal.cwu.edu/` for exam time, and final grades
Other dates:	`http://www.cwu.edu/~regi/course_information.html`

Math 398 is an introduction to Enumerative Combinatorics. The course will focus on the following problem: Given a sequence $S_1, S_2, S_3, \ldots$ of sets, count the number $f(n)$ of elements of S_n "simultaneously." For example, if S_n is the set of *permutations* the set $\{1, 2, \ldots, n\}$ then $f(n) = n!$, and if S_n is the set of *subsets* of the set $\{1, 2, \ldots, n\}$, then $f(n) = 2^n$. There is a philosophical issue of what is meant by simultaneously determining $f(n)$. For example, $f(n)$ may be determined by an *algorithm*, by a *recurrence relation*, by a *closed formula*, or by a *generating function*. Some answers are more useful for certain purposes than others.

The first step towards understanding an algebraic object is to *count* the number of different objects, or to systematically *enumerate* them. We shall look at basic algebraic objects such as sets, permutations, partitions, and multi-sets (unordered sets that may have repeated elements). Order is important for permutations, but not for subsets, repetition is allowed for multisets, but not for subsets. Depending on the problem at hand, you may need to count unordered partitions, or maybe ordered partitions, or maybe partitions with parts of different sizes, or partitions with parts of even size.

Enumerative Combinatorics has applications to many areas including: finite probability, geometry, group theory, and graph theory. It is a basic course in both Mathematics and Computer Science. Most students will have encountered a little Combinatorics in other courses. We shall systematically study counting techniques: addition and subtraction principles, multiplication and division principles, the pigeonhole principle, the inclusion-exclusion principle etc. Solving problems will be a focus for this course. Our strategy will be to bijectively convert our given problem to a standard problem that we know how to solve. For example, every subset of the set $\{1, 2, \ldots, n\}$ can be viewed as a function $f\colon \{1, 2, \ldots, n\} \to \{0, 1\}$ (and conversely) and we can systematically count the functions. The more problems you solve the better you will become at translating to a standard counting problem.

The textbook above is not required. (It is available only as a hardback. Prices on `amazon.com` vary from $122 to $34!) I shall use the textbook for homework and exam problems. Selected portions of the text will be scanned, and can be downloaded from the library. If you do buy the text, however, you will not regret it – it is very well written. As always I recommend reading the relevant section of the text *before* coming to lectures. We will make Tuesday a **problem-solving** day. So on Tuesday bring your printout from the library and your solutions to the homework problems. Warning: Do not turn up on Tuesday without having tried the problems: solving a homework problem for the first time in an exam is a recipe for a disaster. Above all, explain your solutions clearly, and write them neatly as if they were exam problems.

Enumerative Combinatorics requires abstract thought. For this reason, students all over the world struggle when they first meet this course. You should devote at least 8 *productive* hours of work per week to this course.

I plan to cover chapters 1, 2, 3 of the text. I should stress though that the lecture notes, not the textbook, form the body of examinable material. I strongly encourage you to read the relevant parts of the textbook *before* attending lectures, review your lecture notes *after* each lecture, and do all the assigned problems! The way to become a good violin player is to practice. To become good at this course (and hence pass) you must practice. You will learn much more doing the exercises yourself than watching an expert solve them for you!

If you are unable to attend a lecture, you should get a copy of the notes from a classmate *who takes good notes*. Consider forming your own study groups: you can learn a lot by explaining solutions to a friend, and by hearing solutions.

After each test I will post adjacent to my office a list of scores and approximate grades, so you can determine your relative position in the class. You should double-check the time of the final exam by using Safari. The exam will be in our assigned classroom.

Students requiring special accommodation, because of a physical or mental disability, should see me in the first week of the course. Also, if you are quite sick or suffer a notable hardship, then please let me know promptly. Claims of lengthy hardship that are disclosed the day before the final exam receive less sympathy. Although the Registrar will notify you of your final grades, you can find out your (unofficial) grades earlier by using Safari.

The course outcomes are: (i) (passing) students learn to think abstractly, laterally, logically and critically, and (ii) students have a reasonable mastery of the key counting techniques, and (iii) students are well versed in bijectively translating a given problem to a standard counting problem.