**Fall 2012**
**Math 475**
**Applied Analysis 1**
**Bouillon 102, 2:00 - 2:50 MWF**

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**Office Hours:** MWF 11:00 - 11:50, TTh 10:00 - 10:50, and by appointment.

**Required Text:** none, although lecture notes will be available at
`www.cwu.edu/~bisgardj/teaching.html`

# 1 Grades/Exams/Homework

- Grades

  Grades will be calculated using the following weighting system:
  Homework: 50%;
  Exams: 50% total, broken up as follows: 25% for the mid-term and 25% for the final.

- Homework

  You'll get a homework assignment every Wednesday (except the first week!) which will be due on the Thursday of the following week. You may use/ask/talk to whatever/whoever you'd like, as long as you say what resources you used. However, you must write your solutions up in **your own words** to hand in. **DO NOT** wait until the day before homework is due to start working on it. There won't be a large number of problems, but they will take some time to do!

- Exams

  There will be two exams: a mid-term and a final. They will be take-home exams, and you'll have a week to work on them. In contrast to the homework,

  **YOU MUST WORK ON YOUR EXAMS BY YOURSELF!!!!!!!!!!!!!**.

  The mid-term will be handed towards the end of October, and the Final Exam will be handed out on November 26 and be due Wednesday, December 7.

- Expectation for Homework and Exams

  Your homework and exams should be written up neatly and legibly, using complete sentences where appropriate. You should also use correct and proper english - don't expect to get a perfect if your grammar and/or spelling is poor! Writing mathematics well is very difficult, and it takes a lot of practice. It is often helpful to actually read your writing out loud to yourself, and make changes as appropriate.

# 2 Course Goals

There are a number of goals. As an over-arching and rather nebulous goal, we hope to increase your mathematical maturity so that you will be able to read and understand mathematical proofs and ideas in a variety of contexts. You will also practice your proof writing skills. As much more specific goals, we have the following list:

1. Introduction to Analysis on $\mathbb{R}$
   (a) Basic bits of logic (if ... then ... statements, quantifiers).
   (b) Algebraic Properties of $\mathbb{R}$.
   (c) Order properties of $\mathbb{R}$.
   (d) What distinguishes $\mathbb{R}$ from $\mathbb{Q}$? The least upper bound principle! Discuss the distinction between sup and max.
   (e) Definition of convergence of a sequence of real numbers.
   (f) Use least upper bound principle to show that $\lim_{n\to\infty} \frac{1}{n} = 0$.
   (g) The standard properties of sequences.
   (h) Definition of continuity in terms of sequences.
   (i) Properties of continuous functions
2. Vector Spaces and Analysis
   (a) Measuring distance on $\mathbb{R}^3$.
   (b) Definition of a norm and emphasis on how norm structure and vector space structure interact (triangle inequality and homogeneity).
   (c) examples of different norms on $\mathbb{R}^3$, $\mathbb{R}^n$.
   (d) Definition of convergence for a norm, examples.
   (e) Definition of continuity for $f : \mathbb{R}^n \to \mathbb{R}$.
   (f) Open and closed sets.
   (g) Bounded sets.
   (h) Compact sets and why they're so important: continuous functions always have maximizers and minimizers on a compact set!
   (i) Inner products on $\mathbb{R}^n$, and the finite-dimensional version of Riesz Representation Theorem.
   (j) Definition of differentiability for $f : \mathbb{R}^n \to \mathbb{R}$.
   (k) Minimization and applications:
      i. Show that if $A$ is symmetric, then there is a basis for $\mathbb{R}^n$ that consists of orthonormal eigenvectors of $A$.
      ii. Suppose that $A$ is symmetric and $\langle A\mathbf{x}, \mathbf{x}\rangle > 0$ for all $\mathbf{x} \in \mathbb{R}^n \setminus \{\mathbf{0}\}$. Give method of steepest descent and conjugate gradient method for approximating solutions of $A\mathbf{x} = \mathbf{b}$.
3. Critical Points and Flows
   (a) What is a flow? Visualize in $\mathbb{R}^2$ for $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$. (Hopefully, students will have seen this in Math 376/377 by this point!)
   (b) The negative gradient flow associated with a function.
   (c) The mountain pass theorem (and perhaps saddle points?)
4. Functional Analysis
   (a) Introduce $\ell^p$ as a version of $\mathbb{R}^\infty$, along with their norms.
   (b) Closed and bounded sets **ARE NOT** compact in such spaces!
   (c) Weak convergence in $\ell^2$.

(d) weak compactness.

(e) Fourier series on $[-\pi, \pi]$, and how they turn a vector space of functions into $\ell^2$. Comparison to changing bases on $\mathbb{R}^n$, and in particular changing from standard basis to an arbitrary orthonormal basis.

(f) Try to reproduce the appropriate versions of minimization: Show that if $A$ is symmetric and compact, then there is a basis for $\ell^2$ that consists of orthonormal eigenvectors of $A$.

(g) The Fourier type functions give a basis for $L^2[-\pi, \pi]$.

Be aware that we will certainly not cover all of these in a single quarter!

## 3 Legalese/Fine Print

*Students with disabilities who wish to set up academic adjustments in this class should give me a copy of their "Confirmation of Eligibility for Academic Adjustments" from the Disability Support Services Office as soon as possible so we can discuss how the approved adjustments will be implemented in this class. Students without this form should contact the Disability Support Services Office, Bouillon 205 or dssrecept@cwu.edu or 963-2171.*