Math 417A  Loss Models I  
Fall 2013  
Hebeler 112, 1:00 – 1:50 (M, W, F)  

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Office hours: M-Th 2-2:50; and by appointments  
Webpage: www.cwu.edu/~chueh  

Prerequisite: Math 411A or by permission.  

Text:  
2. Recommended: Samuel A. Broverman, Actex C/4 Study Manual  

The students will be introduced to useful frequency and severity models. They will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The students should be able to:  
(i) analyze data from an application in a business context;  
(ii) determine a suitable model including parameter values; and  
(iii) provide measures of confidence for decisions based upon the model.  
The students will be introduced to a variety of tools for the calibration and evaluation of the models.  

Learning outcomes:  
After taking this sequence, the students are expected to be familiar with survival, severity, frequency and aggregate probability models, and use statistical methods to estimate parameters of such models given sample data. The students are further expected to identify steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.  

Course outlines:  
Probability Review  
Discrete random variable: uniform, binomial, negative binomial, Poisson  
Continuous random variable: uniform, normal, gamma  
Multivariable probability: joint distribution, marginal distribution, conditional distribution, independent, covariance, double expectation theorem, sum of independent variables  
Claim Frequency Models  
Binomial, Poisson, negative binomial, geometric distribution  
Creating new counting models  
Mixture models  
Counting process  
Claim severity Models  
Pareto distribution  
Creating new distribution  
Transforming loss random variables for insurance applications  
Discrete distribution formula  
The collective model

LEARNING OUTCOMES for SOA Exam C (Construction and Evaluation of Actuarial Models)  
The candidate is expected to be familiar with survival, severity, frequency and aggregate models, and use statistical methods to estimate parameters of such models given sample data. The candidate is further expected to identify
steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

Specifically, the candidate is expected to be able to perform the tasks listed below. Items in italic font are additions or replacements with respect to the June 2013 syllabus.

Sections A–E have a combined weight of 15-20%.

A. Severity Models
   1. Calculate the basic distributional quantities:
      a) moments
      b) Percentiles
      c) Generating functions
   2. Describe how changes in parameters affect the distribution.
   3. Recognize classes of distributions and their relationships.
   4. Apply the following techniques for creating new families of distributions:
      a) Multiplication by a constant
      b) Raising to a power
      c) Exponentiation,
      d) Mixing
   5. Identify the applications in which each distribution is used and reasons why.
   6. Apply the distribution to an application, given the parameters.
   7. Calculate various measures of tail weight and interpret the results to compare the tail weights.
   8. Identify and describe two extreme value distributions.

B. Frequency Models
   For the Poisson, Mixed Poisson, Binomial, Negative Binomial, Geometric distribution and mixtures thereof:
   1. Describe how changes in parameters affect the distribution,
   2. Calculate moments,
   3. Identify the applications, for which each distribution is used and reasons why,
   4. Apply the distribution to an application given the parameters.
   5. Apply the zero-truncated or zero-modified distribution to an application given the parameters

C. Aggregate Models
   2. Evaluate compound models for aggregate claims.
   3. Compute aggregate claims distributions.

D. For severity, frequency and aggregate models
   1. Evaluate the impacts of coverage modifications:
      a) Deductibles
      b) Limits
      c) Coinsurance
   2. Calculate Loss Elimination Ratios.
   3. Evaluate effects of inflation on losses.

E. Risk Measures
   1. Calculate VaR, and TVaR and explain their use and limitations.

Sections F and G have a combined weight of 20-25%.

F. Construction of Empirical Models
   1. Estimate failure time and loss distributions using:
      a) Kaplan-Meier estimator
      b) Nelson-Åalen estimator
      c) Kernel density estimators
   2. Estimate the variance of estimators and confidence intervals for failure time and loss distributions.
   3. Apply the following concepts in estimating failure time and loss distribution:
a) Unbiasedness  
b) Consistency  
c) Mean squared error  

G. Estimation of decrement probabilities from large samples  
1. Estimate decrement probabilities using both parametric and nonparametric approaches for both individual and interval data  
2. Approximate the variance of the estimators  

H. Construction and Selection of Parametric Models (25-30%)  
1. Estimate the parameters of failure time and loss distributions using:  
   a) Maximum likelihood  
   b) Method of moments  
   c) Percentile matching  
   d) Bayesian procedures  
2. Estimate the parameters of failure time and loss distributions with censored and/or truncated data using maximum likelihood.  
3. Estimate the variance of estimators and the confidence intervals for the parameters and functions of parameters of failure time and loss distributions.  
4. Apply the following concepts in estimating failure time and loss distributions:  
   a) Unbiasedness  
   b) Asymptotic unbiasedness  
   c) Consistency  
   d) Mean squared error  
   e) Uniform minimum variance estimator  
5. Determine the acceptability of a fitted model and/or compare models using:  
   a) Graphical procedures  
   b) Kolmogorov-Smirnov test  
   c) Anderson-Darling test  
   d) Chi-square goodness-of-fit test  
   e) Likelihood ratio test  
   f) Schwarz Bayesian Criterion  

I. Credibility (20-25%)  
1. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.  
2. Perform Bayesian analysis using both discrete and continuous models.  
3. Apply Bühlmann and Bühlmann-Straub models and understand the relationship of these to the Bayesian model.  
4. Apply conjugate priors in Bayesian analysis and in particular the Poisson-gamma model.  
5. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.  

J. Simulation (5-10%)  
1. Simulate both discrete and continuous random variables using the inversion method.  
2. Simulate from discrete mixtures, decrement tables, the (a,b,0) class, and the normal and lognormal distributions using methods designed for those distributions  
3. Estimate the number of simulations needed to obtain an estimate with a given error and a given degree of confidence.  
4. Use simulation to determine the p-value for a hypothesis test.  
5. Use the bootstrap method to estimate the mean squared error of an estimator.  
6. Apply simulation methods within the context of actuarial models.
**Course outlines:**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Days</th>
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<tbody>
<tr>
<td>(one day is one-hour)</td>
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<tr>
<td>I Basic distributional quantities</td>
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<tr>
<td>1. Sums of random variables</td>
<td></td>
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<tr>
<td>2. Tail distributions</td>
<td></td>
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<tr>
<td>3. Measures of risk</td>
<td>5</td>
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<tr>
<td>II Characteristics of actuarial models</td>
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<tr>
<td>1. The role of parameters</td>
<td></td>
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<tr>
<td>2. Continuous models</td>
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<tr>
<td>3. Discrete distributions</td>
<td>7</td>
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<tr>
<td>III Frequency and severity with coverage modifications</td>
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<tr>
<td>1. Deductibles</td>
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<td>2. Loss elimination ratio</td>
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<td>3. Effect of inflation</td>
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<td>4. Policy limits</td>
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<td>5. Coinsurance</td>
<td>8</td>
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<tr>
<td>IV Aggregate loss models</td>
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<tr>
<td>1. Aggregate claims</td>
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<tr>
<td>2. Compound models</td>
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<tr>
<td>3. The individual risk model</td>
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<td>4. Compound Poisson approximation</td>
<td>8</td>
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<tr>
<td>TESTING</td>
<td>2</td>
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<td>TOTAL</td>
<td>30</td>
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**Class format:** Mixture of lectures and in-class problem solving. Students present their solutions of the assigned problems and answer any question raised by the instructor and other students. Group discussions to explain concepts and generate project ideas.

**Grading policy:**

1. Class participation and presentations (50 points)
2. Assignments/Project abstract (50 points)
3. Two tests (100 points)
4. Final exam (50 points)
Final grades will be assigned according to the following scale:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Percentage Range</th>
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<tbody>
<tr>
<td>A</td>
<td>100-93%</td>
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<tr>
<td>A-</td>
<td>92.9-90%</td>
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<tr>
<td>B+</td>
<td>89.9-87%</td>
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<tr>
<td>B</td>
<td>86.9-83%</td>
</tr>
<tr>
<td>B-</td>
<td>82.9-80%</td>
</tr>
<tr>
<td>C+</td>
<td>79.9-77%</td>
</tr>
<tr>
<td>C</td>
<td>76.9-73%</td>
</tr>
<tr>
<td>D+</td>
<td>69.9-67%</td>
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<tr>
<td>D</td>
<td>66.9-63%</td>
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<tr>
<td>D-</td>
<td>62.9-60%</td>
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<tr>
<td>F</td>
<td>59.9% and below</td>
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**Tentative Schedule** (Any change will be announced in class.)

<table>
<thead>
<tr>
<th>Week</th>
<th>Reading Assignment</th>
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<tbody>
<tr>
<td>0. 9/25-9/27</td>
<td>Chapter 3</td>
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<tr>
<td>1. 9/30-10/4</td>
<td>Chapter 4</td>
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<tr>
<td>2. 10/7-10/11</td>
<td>Chapter 5</td>
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<tr>
<td>3. 10/14-10/18</td>
<td>Chapter 5</td>
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<tr>
<td>4. 10/21-10/25</td>
<td>Review</td>
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<tr>
<td></td>
<td><strong>Test 1</strong></td>
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<tr>
<td>5. 10/28-11/01</td>
<td>Chapter 6</td>
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<td>6. 11/04-11/08</td>
<td>Chapter 8</td>
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<tr>
<td>7. 11/11-11/15</td>
<td>Chapter 9</td>
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<tr>
<td>8. 11/18-11/22</td>
<td>Review</td>
</tr>
<tr>
<td></td>
<td><strong>Test 2</strong></td>
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<tr>
<td>9. 11/25-11/29</td>
<td>Chapter 9</td>
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<tr>
<td>11/27-11/29</td>
<td><strong>THANKSGIVING!!</strong></td>
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<tr>
<td>10. 12/02-12/06</td>
<td>Review</td>
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<td></td>
<td><strong>Research Project Title and Abstract due</strong></td>
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<tr>
<td>11. 12/09-12/12</td>
<td><strong>Final Exam (accumulative, higher weight on later material)</strong></td>
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The **Final Exam** will be according to the university schedule.