

**Math 417A Loss Models I**  
 Fall 2015  
 Black Hall 134, 11:00 – 11:50 (M, W, F)

**Instructor:** Dr. Yvonne Chueh

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**Prerequisite:** Math 411A or by permission.

**Text:**

1. Required: Stuart A. Klugman, Harry H. Panjer, Gordon E. Willmot, Loss Models, Wiley 2012.
2. Required: Samuel A. Broverman, Actex Exam C Study Manual, Fall 2014.

The students will be introduced to useful frequency and severity models. They will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The students should be able to:

- (i) analyze data from an application in a business context;
- (ii) determine a suitable model including parameter values; and
- (iii) provide measures of confidence for decisions based upon the model.

The students will be introduced to a variety of tools for the calibration and evaluation of the models.

**Learning outcomes:**

After taking this sequence, the students are expected to be familiar with survival, severity, frequency and aggregate probability models, and use statistical methods to estimate parameters of such models given sample data. The students are further expected to identify steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

**Course outlines:**

Probability Review

Discrete random variable: uniform, binomial, negative binomial, Poisson

Continuous random variable: uniform, normal, gamma

Multivariable probability: joint distribution, marginal distribution, conditional distribution, independent, covariance, double expectation theorem, sum of independent variables

Claim Frequency Models

Binomial, Poisson, negative binomial, geometric distribution

Creating new counting models

Mixture models

Counting process

Claim severity Models

Pareto distribution

Creating new distribution

Transforming loss random variables for insurance applications

Discrete distribution formula

The collective model

**LEARNING OUTCOMES for SOA Exam C (Construction and Evaluation of Actuarial Models)**

The candidate is expected to be familiar with survival, severity, frequency and aggregate models, and use statistical methods to estimate parameters of such models given sample data. The candidate is further expected to identify

steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

Specifically, the candidate is expected to be able to perform the tasks listed below. Items in italic font are additions or replacements with respect to the June 2013 syllabus.

Sections A–E have a combined weight of 15-20%.

#### A. Severity Models

1. Calculate the basic distributional quantities:
  - a) moments
  - b) Percentiles
  - c) Generating functions
2. Describe how changes in parameters affect the distribution.
3. Recognize classes of distributions and their relationships.
4. Apply the following techniques for creating new families of distributions:

##### a) Multiplication by a constant

- b) Raising to a power
- c) Exponentiation,
- d) Mixing

5. Identify the applications in which each distribution is used and reasons why.
6. Apply the distribution to an application, given the parameters.
7. Calculate various measures of tail weight and interpret the results to compare the tail weights.
8. *Identify and describe two extreme value distributions.*

#### B. Frequency Models

For the Poisson, Mixed Poisson, Binomial, Negative Binomial, Geometric distribution and mixtures thereof:

1. Describe how changes in parameters affect the distribution,
2. Calculate moments,
3. Identify the applications, for which each distribution is used and reasons why,
4. Apply the distribution to an application given the parameters.
5. Apply the zero-truncated or zero-modified distribution to an application given the parameters

#### C. Aggregate Models

1. Compute relevant parameters and statistics for collective risk models.
2. Evaluate compound models for aggregate claims.
3. Compute aggregate claims distributions.

#### D. For severity, frequency and aggregate models

1. Evaluate the impacts of coverage modifications:
  - a) Deductibles
  - b) Limits
  - c) Coinsurance
2. Calculate Loss Elimination Ratios.
3. Evaluate effects of inflation on losses.

#### E. Risk Measures

1. Calculate VaR, and TVaR and explain their use and limitations.

Sections F and G have a combined weight of 20-25%.

#### F. Construction of Empirical Models

1. Estimate failure time and loss distributions using:
  - a) Kaplan-Meier estimator
  - b) Nelson-Åalen estimator
  - c) Kernel density estimators
2. Estimate the variance of estimators and confidence intervals for failure time and loss distributions.
3. Apply the following concepts in estimating failure time and loss distribution:

- a) Unbiasedness
- b) Consistency
- c) Mean squared error

G. *Estimation of decrement probabilities from large samples*

1. *Estimate decrement probabilities using both parametric and nonparametric approaches for both individual and interval data*
2. *Approximate the variance of the estimators*

H. Construction and Selection of Parametric Models (25-30%)

1. Estimate the parameters of failure time and loss distributions using:
  - a) Maximum likelihood
  - b) Method of moments
  - c) Percentile matching
  - d) Bayesian procedures
2. Estimate the parameters of failure time and loss distributions with censored and/or truncated data using maximum likelihood.
3. Estimate the variance of estimators and the confidence intervals for the parameters and functions of parameters of failure time and loss distributions.
4. Apply the following concepts in estimating failure time and loss distributions:
  - a) Unbiasedness
  - b) Asymptotic unbiasedness
  - c) Consistency
  - d) Mean squared error
  - e) Uniform minimum variance estimator
5. Determine the acceptability of a fitted model and/or compare models using:
  - a) Graphical procedures
  - b) Kolmogorov-Smirnov test
  - c) Anderson-Darling test
  - d) Chi-square goodness-of-fit test
  - e) Likelihood ratio test
  - f) Schwarz Bayesian Criterion

I. Credibility (20-25%)

1. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.
2. Perform Bayesian analysis using both discrete and continuous models.
3. Apply Bühlmann and Bühlmann-Straub models and understand the relationship of these to the Bayesian model.
4. Apply conjugate priors in Bayesian analysis and in particular the Poisson-gamma model.
5. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.

J. Simulation (5-10%)

1. Simulate both discrete and continuous random variables using the inversion method.
2. *Simulate from discrete mixtures, decrement tables, the  $(a,b,0)$  class, and the normal and lognormal distributions using methods designed for those distributions*
3. Estimate the number of simulations needed to obtain an estimate with a given error and a given degree of confidence.
4. Use simulation to determine the p-value for a hypothesis test.
5. Use the bootstrap method to estimate the mean squared error of an estimator.
6. Apply simulation methods within the context of actuarial models.

**Course outlines:**

<u>Topic</u>	<u>Days</u> (one day is one-hour)
I      Basic distributional quantities 1. Sums of random variables 2. Tail distributions 3. Measures of risk	5
II     Characteristics of actuarial models 1. The role of parameters 2. Continuous models 3. Discrete distributions	7
III    Frequency and severity with coverage modifications 1. Deductibles 2. Loss elimination ratio 3. Effect of inflation 4. Policy limits 5. Coinsurance	8
IV    Aggregate loss models 1. Aggregate claims 2. Compound models 3. The individual risk model 4. Compound Poisson approximation	8
TESTING	2
TOTAL	30

**Class format:** Mixture of lectures and in-class problem solving. Students present their solutions of the assigned problems and answer any question raised by the instructor and other students. Group discussions to explain concepts and generate project ideas.

**Grading policy:**

1. Class participation and presentations (50 points)
2. Assignments/Project abstract (50 points)
3. Two tests (100 points)
4. Final exam (50 points)

Final grades will be assigned according to the following scale:

<b>A</b> 100-93%	<b>A-</b> 92.9-90%	
<b>B+</b> 89.9-87%	<b>B</b> 86.9-83%	<b>B-</b> 82.9-80%
<b>C+</b> 79.9-77%	<b>C</b> 76.9-73%	<b>C-</b> 72.9-70%
<b>D+</b> 69.9-67%	<b>D</b> 66.9-63%	<b>D-</b> 62.9-60%
<b>F</b> 59.9% and below		

### Class Expectations

- **Think critically.** This course will require critical thinking. People that analyze, infer, evaluate, and make reasoned judgments do better in college, make better daily decisions, and have greater professional success. Developing critical thinking should be a key goal of every student.
- **Apply yourself.** This course will take a lot of time and energy. If you have high learning expectations, that is what you will achieve. Success in this course will require significant effort (several hours of study time for each hour of class). Depending on your mathematical background, you may need to spend more or less study time. Attend class regularly, be on time, and budget your time to accommodate the workload.
- **Ask questions.** Loss Modeling is fascinating, but it can be confusing, too. Ask questions. If you aren't clear on something, there are likely others who are equally unclear on the topic.
- **Be informed.** People sometimes use information to manipulate others' behaviors and decision-making in ways not always to your benefit. If you don't understand the mathematical basis of a claim about data, you can't make an informed decision about it. Be curious; try and find out all you can about a topic before you make a decision that may profoundly affect your life.
- **Be respectful.** We will discuss some sensitive and controversial issues in this course. Everyone will respect others' right to express their opinions even if you disagree. Respectful discourse is a minimal expectation of every student.
- **Communicate clearly.** Effective written and oral communication indicates an intelligent mind. Clarity, proper format, spelling, and grammar are expected of every student.
- **Use common sense.** Cheating on assignments or exams, plagiarizing others' work, and turning in late assignments is unacceptable. Any infractions may result in a zero for the assignment, a failing course grade, and the possibility of disciplinary action by the university. I won't accept *anything* late unless you have written documentation from an appropriate source or have made prior arrangements with me. If you have a problem that prohibits you from turning something in on time, let me know ahead of time. In all instances, communicate with me so we can prevent problems.

**Tentative Schedule** (Any change will be announced in class.)

<u>Week</u>	<u>Reading Assignment</u>
0. 9/23-9/25	Chapter 3
1. 9/28-10/2	Chapter 4

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|-----------------|---|
| 2. 10/5-10/9    | Chapter 5   |
| 3. 10/12-10/16  | Chapter 5   |
| 4. 10/19-10/23  | Review<br><b>Test 1</b>   |
| 5. 10/26-10/30  | Chapter 6   |
| 6. 11/02-11/06  | Chapter 8   |
| 7. 11/9-11/13   | Chapter 9   |
| 8. 11/16-11/20  | Review<br><b>Test 2</b>   |
| 9. 11/23- 11/27 | Chapter 9   |
|                 | <i>THANKSGIVING!!</i>   |
| 10. 11/30-12/04 | Review<br><b>Research Project Title and Abstract due</b>          |
| 11. 12/07-12/10 | <b>Final Exam (accumulative, higher weight on later material)</b> |

The <b>Final Exam</b> will be according to the university schedule.
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