Department of Mathematics
Central Washington University

**Math 419B Actuarial Mathematics I**
**Winter 2023**
**SAMU 149, 1:00 – 1:50 P.M., (M,T,W,R)**

**Instructor:** Dr. Yvonne Chueh
**Office:** SAMU 218 I **Phone**: 963-2124
**E-mail:** chueh@cwu.edu
**Office Hours**: M-F 10:00-10:50, and by appointment. You are *always* most *welcome* and *encouraged* to contact me for a question or come to my office hour.

**Prerequisite:** Math 419A and permission.

**Text**:

1. *Actuarial Mathematics for Life Contingent Risks, 2nd Edition*, 2013, Dickson, D., Hardy, M., Waters, H., Cambridge University Press, ISBN: 978-1-110704-407-4. Exercises are considered part of the required readings.
   Math 419A Chapters 1-4; Entire Math 419 sequence covers Chapters 1-13 (Except Section 11.5)
2. Actex manual for Exam FAM-L

**Supplementary Text:**

Life Contingencies: The Mathematics, Statistics, and Economics of Life Insurance
***An open text authored by the Actuarial Community***
**https://openacttextdev.github.io/LifeCon/**

**Optional Spreadsheet Project:** You are given an opportunity to develop an Excel spreadsheet application that will implement the Net Single Premium formulas for a variety of life insurance and life annuity contracts you have learned in math 419A. It will be a valuable tool for single life contracts (this winter quarter) and joint lives can be added in spring quarter. You should use the standard life table (from the FAM-L exam) as a starting point since this table provides the whole life cases for every integer age on the table, thus serves a nice verification vehicle for the formulas built in the cells. You will edit formulas for term life, whole life, and deferred life and make the calculation results accessible to your program users. This project connects real-world work with your computational and conceptual techniques. As an option, you may elect to include the valuation dates at fractional age under various mortality assumptions (UDD, De Moivre, Constant Force, ..etc). Over and above the intensive programming work above, you should come up with a theme title (or "overarching question") for your own spreadsheet project.
The best part of student projects often lies on the learning experience and the creativity originated by yourself or team work! You will earn a great sense of achievement from your work. You are urged to present your finished project to SOURCE, a once-a-year campus symposium event. My role would be to mentor you to handle errors or hurdles in formula comprehensions and coding. Presenting at the SOURCE could be a significant time commitment but surely worthwhile to invest your time for your professional resume according to our alumni and hiring managers.

A possible project expansion for the spring quarter may be on the benefit reserves. You may use life tables your pick from the SOA, AAA websites or internet. I can assist you with formatting the text file into the .xls spreadsheet format. Any other ideas from you are welcome and encouraged!

**Grade Assessment:**

| | |
|---|---|
| 1. Professionalism & presentation | (50 points) |
| 2. Participation | (50 points) |
| 3. Actex Chapter Assignments | (100 points) |
| 4. Two Tests | (200 points) |
| 5. Final exam | (100 points) |
| 6. Spreadsheet Project | (Up to 50 bonus points) |
| **Total:** | **550 points/500 points** |

A perfect score on both of the above categories would result in a total of 700 points. Your course grade will be determined by the percentage ***p*** of these points you earn, according the following scale.

A (100-94%) A- (93-90%) B+ (89-87%) B (86-83%) B- (82-80%) C+ (79-77%)
C (76-73%) C- (72-70%) D+ (69-67%) D (66-63%) D- (62-60%) F (59-0%)

**Class format:** Mixture of lectures and in-class problem solving. Students present their solutions of the assigned problems and answer any question raised by the instructor and other students.

**Tentative Schedule** (The Instructor reserves the right to add or drop quizzes as well as other changes necessary to move the progress forward in light of balancing student learning and course objectives. Any change will be announced in Canvas. Please check the Canvas announcement daily. You can set up an email alert to receive all the Canvas announcements daily.)

| **Week** | **Reading Assignment** |
|---|---|
| 0. 1/4-1/6<br>1. 1/9~1/13 | **Premium Calculation**<br>***Text Reading: 6.1, 6.2, 6.3***<br>***Actex Reading: 5.1~5.4***<br>***Actex Exercise 5: TBA in Canvas*** |
| 2. 1/16-1/20 | ***Text reading: 6.4, 6.5***<br>***Actex Reading 5.5~5.8***<br>***Actex Exercise 5: TBA in Canvas*** |
| 3. 1/23-1/27 | **Chapter Review**<br>***Actex Exercise 5: TBA in Canvas*** |

**Test 1 on Monday, 1/30**

4. 1/30-2/3

## Net Premium Reserves

***Text Reading: 6.6, 6.7***
***Actex Reading: 6.1~6.3***
***Actex Exercise 6: TBA in Canvas***

5. 2/6-2/10

***Text Reading: 6.8, 6.9***
***Actex Reading: 6.4, 6.5***
***Actex Exercise 6: TBA in Canvas***

6. 2/13-2/17

**Chapter Review**
***Actex Exercise 6: TBA in Canvas***

**Test 2 on Wednesday, 2/20**

7. 2/20-2/24

## Gross Premium Reserves

***Text Reading: 8.8~8.10, 8.12***
***Actex Reading: 7.1~ 7.4***
***Actex Exercise 7: TBA in Canvas***

8. 2/27-3/3

***Text Reading: 9.2~9.7***
***Actex Reading: 7.5~7.6***
***Actex Exercise 7: TBA in Canvas***

9. 3/6- 3/10

**Chapter Review**

10. 3/13-3/17

**Comprehensive Final Exam**

The **Final Exam** date will be according to the university schedule unless announced otherwise.

**SOA Exam Syllabus**

https://www.soa.org/4af1a8/globalassets/assets/files/edu/2023/2023-03-exam-fam-syllabus.pdf

## Fundamentals of Actuarial Mathematics Exam-Long Term (FAM-L) Math 419A and 419B

The syllabus for the long-term section of the examination develops the candidate's knowledge of the theoretical basis of contingent payment models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability, mathematical statistics and interest theory is assumed.

1. Topic: Insurance Coverages and Retirement Financial Security Programs (2.5-7.5%)

**Learning Objectives**

The Candidate will understand the key features of insurance coverages and retirement financial security programs.

**Learning Outcomes**

The Candidate will be able to:

a) Define and apply the concept of insurable interest.

b) Identify the long-term insurance coverages (life, health), annuities, and defined benefit and defined contribution pension plans.

2. Topic: Mortality Models (7.5-12.5%)

**Learning Objectives**

The Candidate will understand key concepts concerning parametric and non-parametric mortality models for individual lives.

**Learning Outcomes**

The Candidate will be able to:

a) Understand parametric survival models, life tables, and the relationships between them.

b) Given a parametric survival model, calculate survival and mortality probabilities, the force of mortality function, and curtate and complete moments of the future lifetime random variable.

c) Identify and apply standard actuarial notation for future lifetime distributions and moments, including select and ultimate functions.

d) Given a life table, calculate survival and mortality probabilities, the force of mortality function, and curtate and complete moments of the future lifetime random variable, using appropriate fractional age assumptions where necessary.

e) Understand and apply select life tables.

f) Identify common features of population mortality curves.

3. Topic: Parametric and Non-Parametric Estimation (5-10%)

**Learning Objectives**

The Candidate will understand and be able to estimate parameters for parametric and non-parametric survival models.

**Learning Outcomes**

The Candidate will be able to:

a) Use Maximum Likelihood Estimation to estimate log-likelihood functions for various laws of mortality

b) Apply Kaplan Meier and Nelson Aalen methods to estimate empirical survival functions using censored and truncated lifetime data.

c) Calculate approximate standard errors of the parameter/probability estimates.

d) Construct linear and non-linear confidence intervals (as appropriate) for parameters/estimates.

4. Topic: Present Value Random Variables for Long-Term Insurance Coverages (10-15%)

**Learning Objectives**

The Candidate will be able to perform calculations on the present value random variables associated with benefits and expenses for long term insurance coverages.

**Learning Outcomes**

The Candidate will be able to:

a) Identify the present value random variables associated with life insurance, endowment, and annuity payments for single lives, based on annual, 1/m-thly and continuous payment frequency.

b) Calculate probabilities, means, variances and covariances for the random variables in Topic 10(a), using fractional age or claims acceleration approximations where appropriate.

c) Understand the relationships between the insurance, endowment, and annuity present value random variables in Topic 10(a), and between their expected values.

d) Calculate the effect of changes in underlying assumptions (e.g., mortality and interest).

e) Identify and apply standard actuarial notation for the expected values of the random variables in Topic 10(a).

5. Topic: Premium and Policy Value Calculation for Long-Term Insurance Coverages (15-20%)

**Learning Objectives**

The Candidate will be able to use and explain the premium and policy value calculation processes for long-term insurance coverages.

**Learning Outcomes**

The Candidate will be able to:

a) Identify the future loss random variables associated with whole life, term life, and endowment insurance, and with term and whole life annuities, on single lives.

b) Calculate premiums based on the equivalence principle, the portfolio percentile principle, and for a given expected present value of profit, for the policies in Topic 11(a).

c) Calculate and interpret gross premium, net premium and modified net premium policy values for the policies in Topic 11(a).

d) Calculate the effect of changes in underlying assumptions (e.g., mortality and interest).

e) Apply the following methods for modelling extra risk: age rating; constant addition to the force of mortality, constant multiple of the rate of mortality.

Readings - Long-Term (FAM-L):

Actuarial Mathematics for Life Contingent Risks, Third Edition Dickson, C.M.D., Hardy, M.R., Waters, H.R. (2020), Cambridge University Press ISBN: 978-1-108-47808-3. Exercises are considered part of the required readings.

Chapter 1
Chapter 2
Chapter 3 (except Sections 3.11,3.12)
Chapter 4
Chapter 5
Chapter 6
Chapter 7 (Sections 1-3 [except 2.4, 2.5], 7, 8)
Chapter 18 (Sections 1-5)

Other Resources - Long-Term (FAM-L):
• Tables for FAM-L
Excel Workbook for FAM-L Tables (These spreadsheets were used to develop the tables used for the FAM-L exam and is provided for educational purposes only. The workbook will not be available at the FAM-L exam.)
• Notation and Terminology used on FAM-L
• Sample Questions and Solutions for FAM-L
Note: The texts, notation and terminology notes, and any study notes will not be available with the examination booklet. A copy of the Tables will be available.

https://www.soa.org/4a645e/globalassets/assets/files/edu/2023/spring/syllabi/2023-07-altam-syllabus.pdf

**Advanced Long-Term Actuarial Mathematics (ALTAM) Exam SPRING 2023 Math 419C**

The ALTAM Exam is a three-hour exam consisting of 60 points of written-answer questions. The exam will be taken at Prometric testing centers. The written-answer questions will be displayed on the computer and answered in a paper answer booklet to be provided and collected by Prometric. Tables will be provided in an Excel workbook, which may also be used for calculations. However only the paper answer booklet will be submitted for grading. Paper versions of the exam and tables will not be provided. Candidates will be provided with a scratchpad.
Knowledge of the FAM Exam material is assumed for the ALTAM Exam.
Exam Registration Candidates may register online or with an application.
Introductory Study Note The Introductory Study Note has a complete listing of all study notes as well as errata and other important information.
Past Exams Past LTAM Exams from October 2018 through present are available on the SOA website. There are no past ALTAM Exams. Updates Candidates should be sure to check the Updates page on the exam home page periodically for additional corrections or notices.

1. Topic: Survival Models for Multiple State Contingent Cashflows (10-20%)
Learning Objectives
The Candidate will understand key concepts concerning multiple state mortality/morbidity and joint life mortality models for insurance and annuity contracts.
Learning Outcomes
The Candidate will be able to:
a) Apply Markov multiple state models to state-contingent life and long-term health insurance benefits, and Continuing Care Retirement Communities (CCRCs).
b) Understand and critique the assumptions underlying Markov multiple state models for long-term insurance.
c) Derive and apply Kolmogorov's forward equations for continuous time Markov multiple state models.
d) Calculate state-dependent probabilities for continuous time Markov models.
e) Apply the Chapman-Kolmogorov equations to calculate discrete time transition probabilities in the Markov model.
f) Construct and deconstruct multiple decrement tables using the associated single decrement models and appropriate fractional age assumptions.
g) Calculate maximum likelihood estimates of transition intensities and probabilities for multiple state and multiple decrement models, assuming piecewise constant transition intensities.
h) Calculate approximate confidence intervals for the estimators in Topic 1(g), using asymptotic properties of the maximum likelihood estimators.

2. Topic: Premium and Policy Valuation for Long-Term State-Dependent Coverages (12-20%)

Learning Objectives
The Candidate will be able to perform calculations on the present value random variables associated with benefits and expenses for single life, joint life, CCRCs, or other state-dependent insurance and annuity contracts.
Learning Outcomes
The Candidate will be able to:
a) Define and interpret state-dependent insurance and annuity present value random variables and identify and calculate their expected values.
b) Derive and apply two-term and three-term Woolhouse approximations for calculating expected present values of state-dependent cash flows.
c) Calculate premiums for state-dependent life insurance, long-term health insurance, and CCRCs using the equivalence principle.
d) Calculate policy values for state-dependent life insurance, long-term health insurance, and CCRCs.
e) Identify and apply Thiele's differential equation in a single life or multiple state setting.

3. Topic: Joint Life Insurance and Annuities (8-16%)
Learning Objectives
The Candidate will understand key concepts concerning estimation and construction of multiple state and joint life models for insurance and annuity contracts.
Learning Outcomes
The Candidate will be able to:
a) Understand how joint-life mortality can be modelled using (i) a time-to-status-failure random variable, and (ii) a multiple state model.
b) Understand the implications of independence or dependence of future lifetimes in both versions of the joint life model from 3(a). Identify sources of dependence and understand how they are accommodated in the models.
c) Calculate premiums for insurance and annuities on joint lives using the equivalence principle.
d) Calculate policy values for insurance and annuities on joint lives.

4. Topic: Profit Analysis (10-20%)
Learning Objectives
The Candidate will be able to analyze emerging surplus, and to set premiums and reserves, using profit testing.
Learning Outcomes
The Candidate will be able to:
a) Calculate and interpret common profit measures such as expected profit, actual profit, gain, gain by source and period, profit signature, profit vector, net present value, internal rate of return, profit margin, and discounted payback period for long-term life and health insurance, and annuity contracts.
b) Calculate premiums for long-term life and health insurance and annuity contracts based on a specified profit objective.
c) Calculate reserves for long-term life and health insurance and annuity contracts using profit testing.
5. Topic: Pension Plans and Retirement Benefits (10-18%)
Learning Objectives
The Candidate will understand how retirement benefits are accrued, valued, and funded.
Learning Outcomes
The Candidate will be able to:
a) Calculate replacement ratios for Defined Contribution (DC), and Defined Benefit (DB) plans, including final average salary (FAS), career average earnings (CAE), and career average revalued earnings (CARE) plans.

b) Calculate the required contribution rate to meet a target replacement ratio for a DC plan participant, using a deterministic approach.
c) Identify, interpret, and apply service table and salary scale functions for pension plan valuation.
d) For a DB plan, calculate and interpret replacement ratios, accrued benefits, including benefits on early exit from the plan.
e) For a DB plan, calculate and interpret the actuarial accrued liability and the normal cost for benefits payable on age retirement or early exit using the projected unit credit (PUC) and traditional unit credit (TUC) valuation methods.
f) Identify and interpret the assumptions and funding methods used for retiree health care valuation.
g) Calculate and interpret the expected present value of future benefits, accumulated postretirement benefit obligation (APBO), and the normal cost or service cost for retiree health care plans.

6. Topic: Universal Life Insurance (10-18%)
Learning Objectives
The Candidate will understand the policy design and benefits payable under Type A and Type B Universal Life contracts and be able to assess and quantify account values, premiums, and reserves.
Learning Outcomes
The Candidate will be able to:
a) Understand the cashflows and calculate account values and benefits under Type A and Type B Universal Life policies.
b) Calculate reserves for no-lapse guarantees.
c) Use deterministic profit testing to calculate premiums and to assess emerging surplus for Universal Life insurance.

7. Topic: Embedded Options in Life Insurance and Annuity Products (10-18%)
Learning Objectives
The Candidate will understand various types of equity-linked life insurance guarantees, options that are embedded in life insurance and annuity contracts and be able to price, reserve, and hedge the risk inherent in these options.
Learning Outcomes
The Candidate will be able to:
a) Define and calculate payoffs under each of the following options embedded in insurance and annuity contracts:
• Guaranteed minimum death benefit
• Guaranteed minimum maturity benefit
• Guaranteed minimum income benefit
• Guaranteed minimum withdrawal benefit
b) Value the following options embedded in insurance and annuity contracts, using the Black-Scholes model:
• Guaranteed minimum death benefit
• Guaranteed minimum accumulation/maturity benefit
c) Construct a replicating portfolio for the options in 7(b) using delta-hedging
d) Understand and evaluate the costs associated with discrete-time rebalancing.

Resources
• Actuarial Mathematics for Life Contingent Risks, Third Edition Dickson, C.M.D., Hardy, M.R., Waters, H.R. (2020), Cambridge University Press ISBN: 978-1-108-47808-3. Exercises are considered part of the required readings.

o Chapter 7 (Sections 2.4 and 4)
o Chapter 8
o Chapter 9
o Chapter 10
o Chapter 11
o Chapter 13 (except Section 8)
o Chapter 14
o Chapter 15 (Sections 1-3)
o Chapter 17
o Chapter 18 (Section 6)
• Notation and Terminology used on Exam ALTAM (forthcoming)
• Note on VA income benefits (forthcoming)
• Exam ALTAM Tables (forthcoming)
Excel Workbook for Exam ALTAM (forthcoming)
• Formula Sheet (forthcoming)
• Sample questions and solutions (forthcoming)
Note: The text and study notes will not be available with the examination booklet. The Excel Workbook for Exam ALTAM and a pdf copy of the Formula Sheet will be available.

**ACADEMIC POLICIES & PROFESSIONAL PROTOCOL**
The following policies and protocols have been established based on university policies and state and federal laws. They are equally binding for all students enrolled in each of my classes.

**A. Equal Educational Opportunity:**
Central Washington University seeks to provide reasonable accommodations for all qualified individuals with disabilities. Accommodations are intended to minimize the functional limitations of a disability and provide the student equal access to the educational process. Please inform me how I might support you in this regard.

**B. Discrimination, Intimidation, & Harassment:**
Hate speak and racist or sexist dialogue and behaviors will not be tolerated. The right of all students to equal access of the course content in an environment free of prejudice, discrimination, and harassment will be respected and upheld. All illegal behavior will be reported to the proper municipal and university authorities.

**C. Professional Participation:**
The nature of the course requires that each student be treated with respect, dignity, and sensitivity. While we can agree to disagree in a professional manner, all students are responsible for actively participating in all classroom activities in a positive, sensitive, and contributory manner. **Each class members' professionalism will be evaluated based on the instructor's discretion.**

Students will also be graded on their active, professional participation**. Please turn off your electronic devices completely during class session.** Be advised you will be marked down **without warning** for playing computer games, checking or looking at mobile phones, watching DVDs, talking on the phone, reading the newspaper, grading papers or projects, studying for other exams, working on other class assignments, engaging in incessant talking of a social nature, and / or (but not limited to) behaviors that disturb the learning environment for other students.

**D. Academic Integrity:**

All cheating, plagiarism and forgery will be referred for disciplinary action and automatically result in an "F" for the course. You are obligated to cite all electronic or bibliographic references for works that are not authored or created by you. Please use the format outlined by the American Psychological Association. If for some reason you do not have a complete reference for a document, do the best you can by providing an author, a date, a workshop site, etc. If a document has been translated, please give credit to the person whose talents made it readable to you or others.

All violations of the student code will be reported to the Department of Mathematics, College of the Sciences, Vice President for Student Affairs & other CWU departments for disciplinary action.